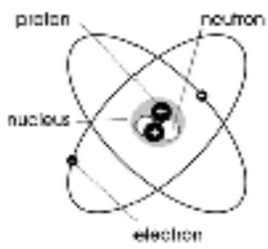
ELECTROSTATIC FIELDS

Electric charge

Ordinary matter is made up of atoms which have positively charged nuclei and negatively charged electrons surrounding them. A body can become charged if it loses or gains electrons. Charge is quantised as a multiple of the electron or proton charge.

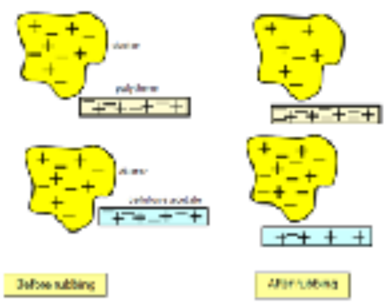


Particle	Charge	Mass
Electron	-1.6×10^{-19} C	$9.1093835 \times 10^{-31}$ kg
Proton	1.6×10^{-19} C	$1.6726219 \times 10^{-27}$ kg
Neutron	0	1.6749×10^{-27} kg

The unit of electric charge is the Coulomb (*C*). The influence of charges is characterised in terms of the forces between them and the electric field and voltage produced by them.

This means that 1 coulomb of charge consists of $(1.6 \times 10^{-19})^{-1}$ electrons or protons. This is equal to 6.24×10^{18} electrons or protons. One Coulomb of charge is the charge which would flow through a 120-watt light bulb in one second. Two charges of one Coulomb each separated by a meter would repel each other with a force of about a million tons!

Charging insulators by friction

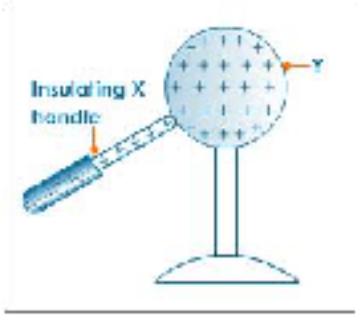


When an insulator is rubbed with a cloth, electrons are transferred to or from the insulator. This excess or deficiency of electrons leave the insulator charged.

Charging conductors by direct contact

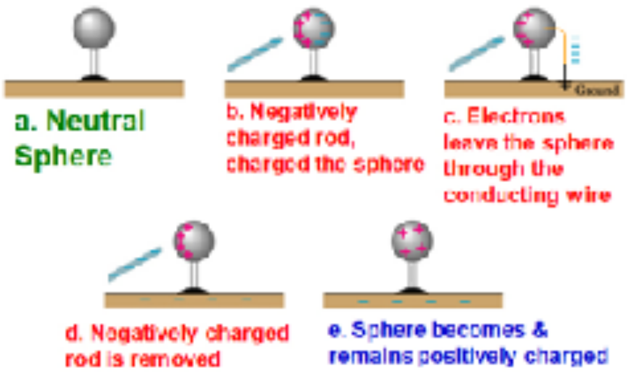
Conductors are materials which contain free electrons. Hence, current can flow through them. Therefore, for them to become charged it is necessary that they are insulated. Otherwise they will be earthed and neutralised.

When an insulated hollow metal sphere becomes in contact with a charged object, it will acquire the same charge as that of the object.



Charging conductors by induction

When an object gets charged by induction, a charge is created by the influence of a charged object but not by contact with a charged object. The word induction means to influence without contact.



Coulomb's law

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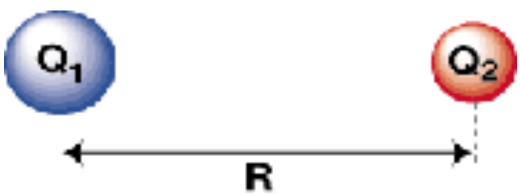
Coulomb's law states that the force between two charged particles is directly proportional to the product of their charges and inversely proportional to the square of their separation.



ε_0: permittivity of free space (or vacuum) and has a value of 8.85×10^{-12} Fm^{-1}.

ε: permittivity of a medium is given by $\varepsilon = \varepsilon_0 \varepsilon_r$; where ε_r is the relative permittivity of the medium and has no units.

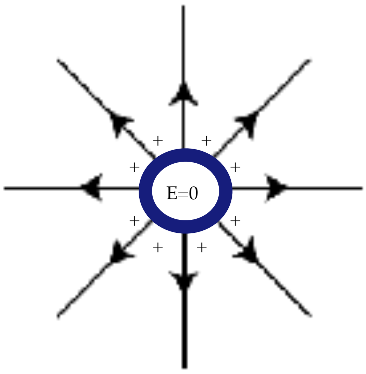
ε_r: (air) = 1

A material with high permittivity is one which reduces appreciably the force between two charges compared with the vacuum value. For example, the relative permittivity of water is about 80. This implies that the magnitude of the force between two charges in water is reduced to about 1/80 of its value in air.

The electric field

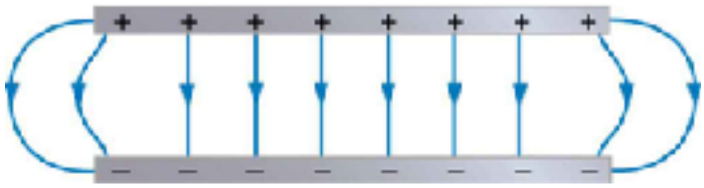
An Electric field is a region in space where an electric charge experiences a force due to another charged object. The direction of the electric field at a point depends on the direction of the force acting on a test charge (unit positive charge) placed at that point. A line of force is the path taken by a free moving test charge. The amount of force lines per unit area represent how strong the field is in that area.

Electrostatic forces can be both attractive and repulsive, depending on the nature of the charge of the particles. In this manner electrostatic forces differ from gravitational forces, which are only attractive.



There is no electric field in the sphere. However, the electric field around the sphere is as if all the charge where at the centre of the sphere.

The parallel equally spaced field lines between the plates indicate that between the field is uniform. However, at the edge, it is no longer so.



Electric field strength (Intensity)

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The electric field strength or electric intensity *E* at a point in an electric field is defined as the force per unit positive charge *q* placed at that point. Its unit is the NC^{-1} or Vm^{-1}.

$$E = \frac{F}{q}$$

The electric field strength at a distance *r* from a point of charge is given by:

$$E = \frac{Q}{4\pi\varepsilon_o r^2} = \frac{Q}{4\pi r^2} \times \frac{1}{\varepsilon_o}$$

The electric field strength is produced by some charge external to the test charge. It is not the field produced by the test charge (q) itself. The electric field strength is a vector quantity. Furthermore, the electric field strength is said to exist at some point regardless whether a test charge is located at that point.

The charge density σ is the charge per unit area. Hence,

$$\sigma = \frac{Q}{A}$$

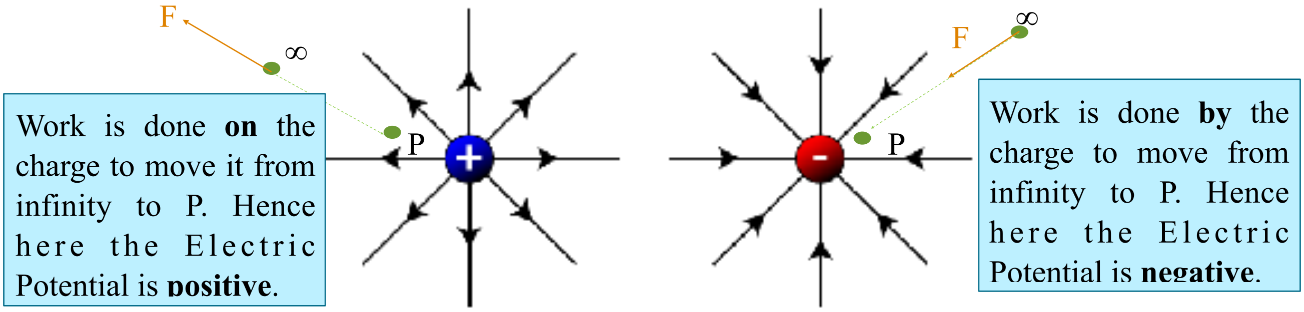
$$\Rightarrow E = \frac{\sigma}{\varepsilon_o}$$

The direction of the electric field strength $\boldsymbol{E}$ depends on the point charge of $\boldsymbol{Q}$. The direction of $\boldsymbol{E}$ follows the direction of the lines of force produced by the point charge.

Electric potential

The potential V at a point in an electric field is numerically equal to the work done in moving a unit positive charge from infinity to that point.

Electric potential isI a scalar quantity and is measured in Volts, V or JC^{-1}. Gravitational potential is always negative since a gravitational field is always attractive in nature. However, electric potential can be either positive or negative.



If W is the work done in moving a small test charge q from infinity to the point, then the potential of the point $\boldsymbol{V}$ is given by:

$$V = \frac{W}{q}$$

Relation between electric field strength and electric potential

The potential at a point a distance r from an isolated point charge Q is given by:

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

This equation shows that the electric potential V_E due to a positive charge is positive while that due to a negative charge is negative.

Consider a positive charge Q at point R as shown above. Let the field intensity at Y be $\boldsymbol{E}$. Suppose that a unit positive charge at Y is moved a very short distance Δx to S. The work done to overcome the repulsion between like charges is given by:

$$\Delta W = F\Delta x$$

$$E = -\frac{F}{q}$$

Note that Since $\boldsymbol{E}$ and $\boldsymbol{F}$ are opposite in direction, then a negative sign is inserted in the equation.

Hence,

$$\Delta W = -Eq\Delta x$$

The potential difference between two points is numerically equal to the work done in moving a unit positive charge between the points.

Therefore,

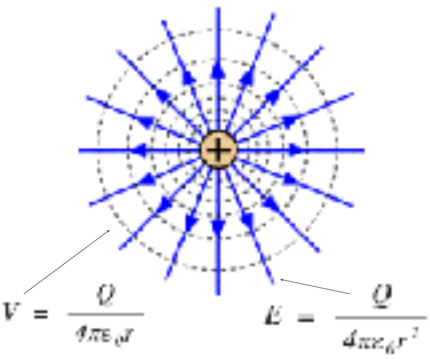
$$\Delta V = \frac{\Delta W}{q} = -\frac{Eq\Delta x}{q}$$

Hence,

$$E = -\frac{\Delta V}{\Delta x}$$

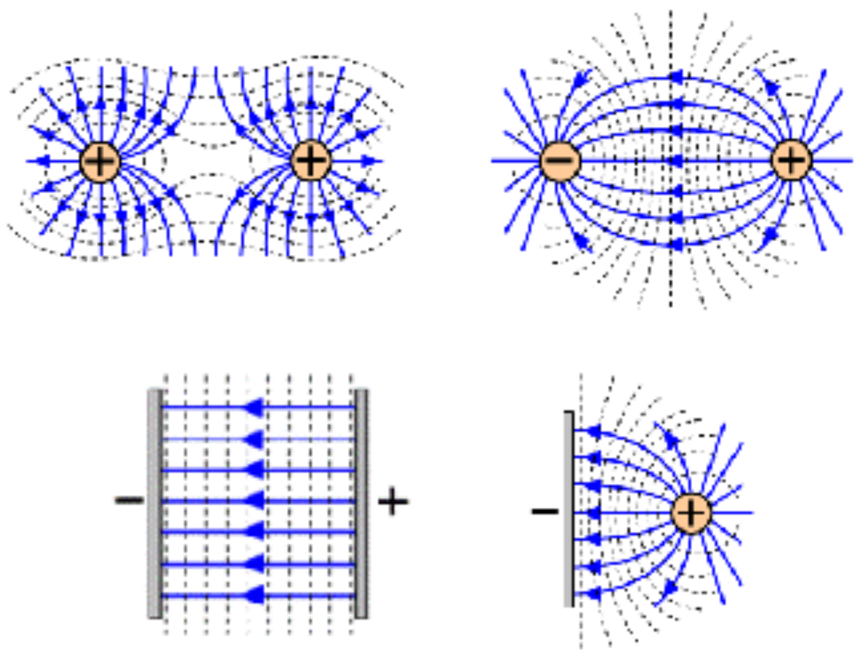
The ratio $\frac{\Delta V}{\Delta x}$ is called the potential gradient.

Equipotential surfaces

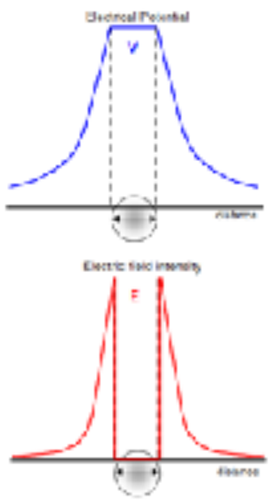


All points in an electric field that have the same potential can be thought of lying on a surface called equipotential surface.

When a point charge moves over an equipotential, no electrical energy change takes place since no work is done. Consequently, the force due to the field must act at right angles to an equipotential surface at any point.



V and E for a charged sphere



From the results of Faraday's ice-pail experiment, irrespective of whether a conductor is hollow or solid, there is no net charge inside it.

The potential V for all points inside a charged solid conducting sphere is the same as at the surface

The electric field intensity E is equal to zero for all points within the surface of a charged solid conducting sphere.

Work done in placing a charge

Suppose that a charge Q_2 is displaced from Y to X. Then:

$$W = \frac{Q_1 Q_2}{4\pi\varepsilon_0}\left(\frac{1}{x} - \frac{1}{y}\right)$$

The work done W in moving a charge Q_2 from infinity to a point at distance x from Q_1 is equal to the electrical potential energy acquired by Q_2 and is given by:

$$W = \frac{Q_1 Q_2}{4\pi\varepsilon_0}\left(\frac{1}{x} - \frac{1}{\infty}\right)$$

$$W = \frac{Q_1 Q_2}{4\pi\varepsilon_0 x}$$

Electric potential difference

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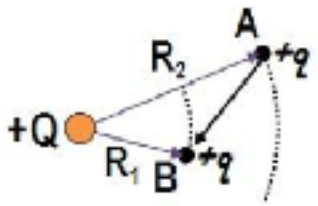
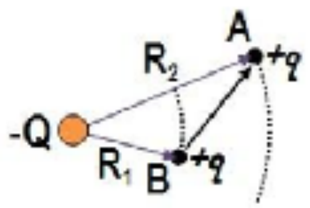


The electrical potential difference of a positively charged particle $+q$ increases when it moves from a point of low potential to a point of higher potential in the field of charge $+Q$ moving from A to B. For a test charge $+q$, in the field of charge $+Q$, moving from A to B:

$$P.D. = V_B - V_A = \frac{Qq}{4\pi\varepsilon}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$



The electrical potential difference of a positively charged particle $+q$ increases when it moves from a point of low potential to a point of higher potential in the field of charge $-Q$ moving from B to A. For a test charge $+q$, in the field of charge $-Q$, moving from B to A:

$$P.D. = V_A - V_B = \frac{Qq}{4\pi\varepsilon}\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$$

The electrical potential difference is the work done per unit charge, on the charge $+q$ by some external agent. It follows that the work done in moving a charge through a potential difference is given by the equation:

$$W = qV$$

W - work done on charge (J)

q - the charge (C)

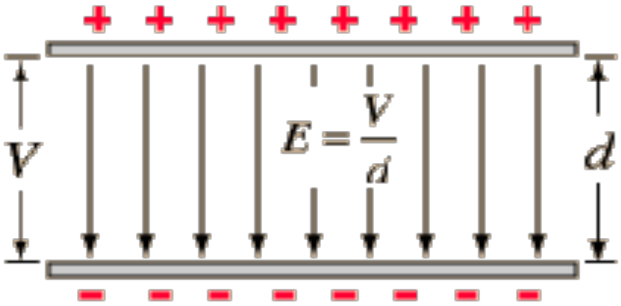
V - the potential difference (V or JC^{-1})

The electric potential difference between two points in an electric field is numerically equal to the work done in moving a unit positive charge from the point of lower potential to that a higher potential.

The unit of p.d. is the Volt (V). It is also defined as if the work done in causing one Coulomb of electric charge to flow between two points is one Joule, then the p.d. between these points is one Volt i.e. $1V=1JC^{-1}$.

Charging parallel plates

Consider the space between two charged plates, distance *d* apart and a potential difference ***V*** across them.



The charges give rise to a uniform electric field. Work *W* is done to move the charge *q* from the negative to the positive plate.

From the definition of electrical potential ***V***, it follows that:

$$W = qV$$

If *F* is the force used to move *q*, then:

$$W = Fd$$

$$\Rightarrow Fd = qV$$

$$\Rightarrow \frac{F}{q} = \frac{V}{d}$$

$$\Rightarrow E = \frac{V}{d}$$

The unit for electric field intensity is Vm^{-1}.

Alternatively, one can say that since the field between the plates is uniform (except at the edges), the potential gradient is the same at any point between the plates.

$$E = -\frac{\Delta V}{\Delta x}$$

The potential gradient at each of these points is equal to the average potential gradient which is equal to $\frac{V}{d}$.

Charged particle in an electric field

Consider charged particles moving in the direction of the electric field or in the opposite direction. The potential difference across the plates $V = V_A - V_B$, since plate A is at a higher potential than B. The field intensity is uniform, hence $E = \frac{V}{d}$.

A positive charge accelerates from a region of high potential (V_A) to a region of lower potential (V_B).

A negative charge accelerates from a region of low potential (V_B) to a region of higher potential (V_A).

When a charged particle ($+q$ or $-q$) and mass m is placed in an electric field E, the electrostatic force F_E exerted on the charge is qE.

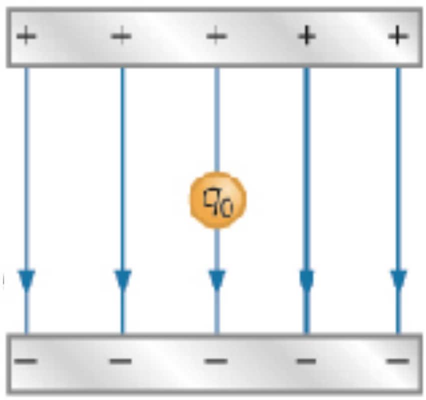
- If q is positive, $\boldsymbol{F}_{\mathrm{E}}$ is in the direction of $\boldsymbol{E}$.
- If q is negative, $\boldsymbol{F}_{\mathrm{E}}$ is in the direction opposite to $\boldsymbol{E}$.

If the space between the charged plates is a vacuum and neglecting any gravitational effects on the charged particles, $\boldsymbol{F}_{\mathrm{E}}$ is the only force on the particle. Hence, $\boldsymbol{F}_{\mathrm{E}}$ is the resultant force acting on the particle causing it to accelerate.

Hence, by Newton's Second Law of motion;

$$\Rightarrow a = \frac{qE}{m}$$

Stationary charged particle in an electric field 9 10 11 12



An x-ray tube ionises the air (into positive ions and electrons) between the plates. Some electrons of total charge q became attached to an oil particle placed between the plates. The p.d. is adjusted until the particle remains stationary. The magnitude of the electric field strength $\boldsymbol{E}$ is given by $E = \frac{V}{d}$.

Upward force $F = Eq = \frac{Vq}{d}$

Downward force $F = mg$

Hence, $\frac{Vq}{d} = mg$

Energy considerations

By the law of conservation of energy:

loss in electrical potential energy = gain in kinetic energy

$$qV = \frac{1}{2}mv^2$$

If an electron is accelerated by an electric field, the energy it gains as it moves through a p.d. of one volt is 1.6×10^{-19} J.

$$\Delta W = e\Delta V$$

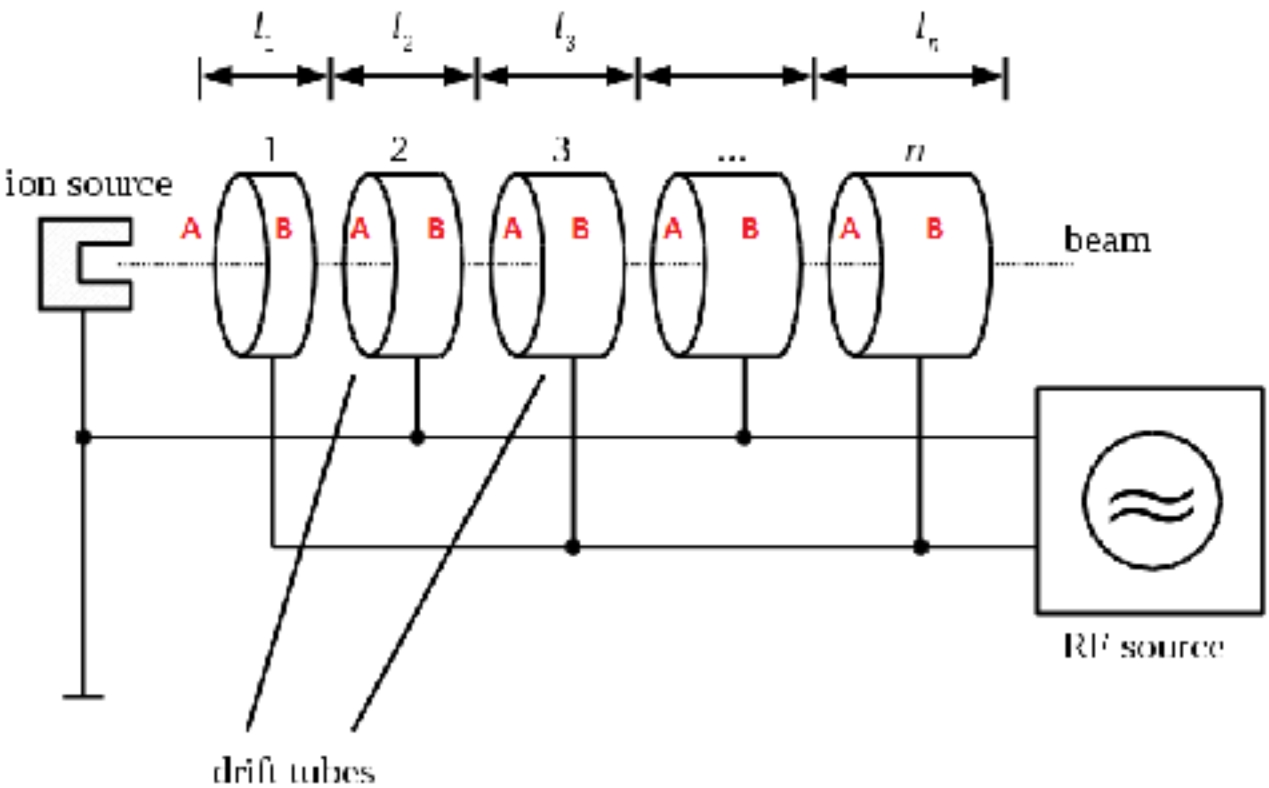
$$\Delta W = 1.6 \times 10^{-19} \times 1$$

$$\Delta W = 1.6 \times 10^{-19} J$$

This quantity is known as an electron volt, eV, and it is often used as a unit of energy when very small values of energy are considered.

$$1eV = 1.6 \times 10^{-19} J$$

Linear accelerators



A linear accelerator (1930) is a type of accelerator that greatly increases the velocity of charged subatomic particles or ions by subjecting the charged particles to a series of oscillating electric potentials along a linear beam line.

Suppose that positive ions travel from the source to electrode 1, where 1 is momentarily at a negative potential. When the ions pass through electrode 1, the a.c. supply reverses and so electrode 1 repels the ions whilst the next electrode, 2 becomes negative and so attracts them.

As the ions pass through electrode 2, the a.c. supply reverses again and so electrode 2 repels the ions and electrode 3 becomes negative and so attracts them. This procedure continues with a number of electrodes and the ions acquire high speed.

As the speed of the ions increases, less time is taken to traverse a fixed distance. To compensate for this, successive electrodes in a linear accelerator increase in thickness gradually. Altogether, electrodes can add up to a distance of about 3km. The energy acquired by particles in some accelerators is of the order of GeV but particles can never reach the speed of light. According to Einstein's theory, the mass of a particle increases with increasing speed and approaches infinity as its speed approaches that of light.

A linear accelerator is most commonly used for external beam radiation treatments. It uses microwave technology to accelerate electrons in a part of the accelerator called the 'wave guide', then allows these electrons to collide with a heavy metal target. As a result of the collisions, high-energy photons are produced from the target.

The high energy x-rays are directed to the patient's tumour and shaped as they exit the machine to conform to the shape of the patient's tumour. Radiation can be delivered to the tumour from any angle by rotating the gantry and moving the treatment couch.

Deflection of charged particles in an electric field

Consider a beam of electrons each moving with a velocity ***u***, entering a uniform electric field of intensity ***E*** which is perpendicular to their direction of motion. Here, we are assuming that the gravitational forces acting on the particle are negligible and that the particles are enclosed in a vacuum at all times.

Once in the field, each electron is subjected to a force $F_e = eE$ in the positive direction of the *y*-axis and therefore by Newton's second law of motion, the electron accelerates with an acceleration of $\frac{eE}{m}$ (*m* is the mass of the electron).

At the instant an electron enters the field, its *y*-component of velocity is zero and therefore after it has spent a time *t* in the field, it will have undergone a vertical displacement ***y***, given by the equation:

$$y = 0 + \frac{1}{2}\left(\frac{eE}{m}\right)t^2$$

$$y = \frac{1}{2}\left(\frac{eE}{m}\right)t^2$$

The velocity of the electrons in the *x*-direction is not influenced by the electric field and hence it moves with constant velocity along the *x*-direction. Hence, the horizontal displacement *x is given by:* $x = ut$

The electron's *y*-component of velocity is affected by the field and therefore:

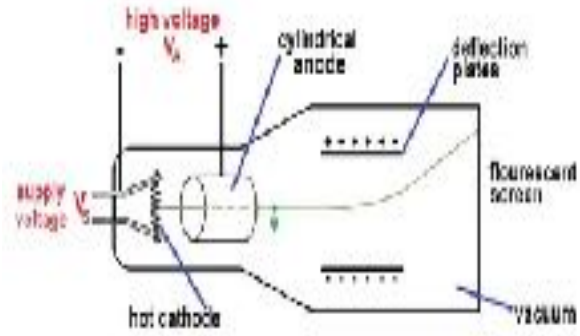
$$v_y = u + a_y t$$

$$v_y = 0 + \left(\frac{eE}{m}\right)t$$

Once the electron has left the field, its path is linear (in a straight line) and since the electron gained velocity in the *y*-direction whilst in the field, its kinetic energy increases. The situation is analogous to that of a body in a uniform gravitational field. In each case the magnitude and the direction of the force are constant.

Electron beams

Cathode Ray tube televisions, oscilloscopes and X-ray tubes all produce fast moving electrons. A low voltage supply heats the filament, causing it to emit electrons. The electrons are accelerated towards the cylindrical anode by a high voltage supply. As the electrons accelerate between the hot cathode and the anode, they lose electrical potential energy and gain kinetic energy.



For electrons of mass m_e and charge e, moving through a p.d. V_A:

Electrical P.E. lost = Gain in K.E.

$$eV_A = \frac{1}{2}m_e v_e^2$$

v_e - velocity of the electron on leaving the cylindrical anode

V_A - p.d. between the hot cathode and the anode

(note: an electron always accelerates from a low potential to a higher potential)